Analysis of N port Power Combiners by Modal Analysis

Introduction

The design and analysis of N-port power combiners is tedious using standard circuit analysis. Of course, numerical analysis by use of modern CAD programs, such as MWO or ADS, greatly simplifies developing answers but does not result in intuitive understanding of the performance. Modal analysis greatly facilitates the design and analysis of many N-port applications allows a more intuitive understanding of how they function. This article explains what modal analysis is and applies it to a number of generic combiner topologies.

Examples of N-Port Circuits

There are numerous generic types of multi-port circuits to which a modal analysis provides an intuitive and analytic approach to designing and understanding. The following figures show some of these:

- Figure 1 shows two types of Wilkinson combiners. The first is a non-isolated combiner and the second is the original isolated version proposed by Ernest J. Wilkinson in 1960. In both versions, the N source impedances are transformed to an equivalent impedance at the junction with the output line such that, the parallel impedance at the output of the input lines is the same as the input impedance of the output line at the junction. Details of the impedance combining are discussed in a separate article.
- Figure 2 shows an isolated multi-section binary Wilkinson in which the isolation occurs at every 2-way split.
- Figure 3 shows a multi-section, N-Way Wilkinson in which a number of sections of resistors are used to achieve isolation over a wide bandwidth.

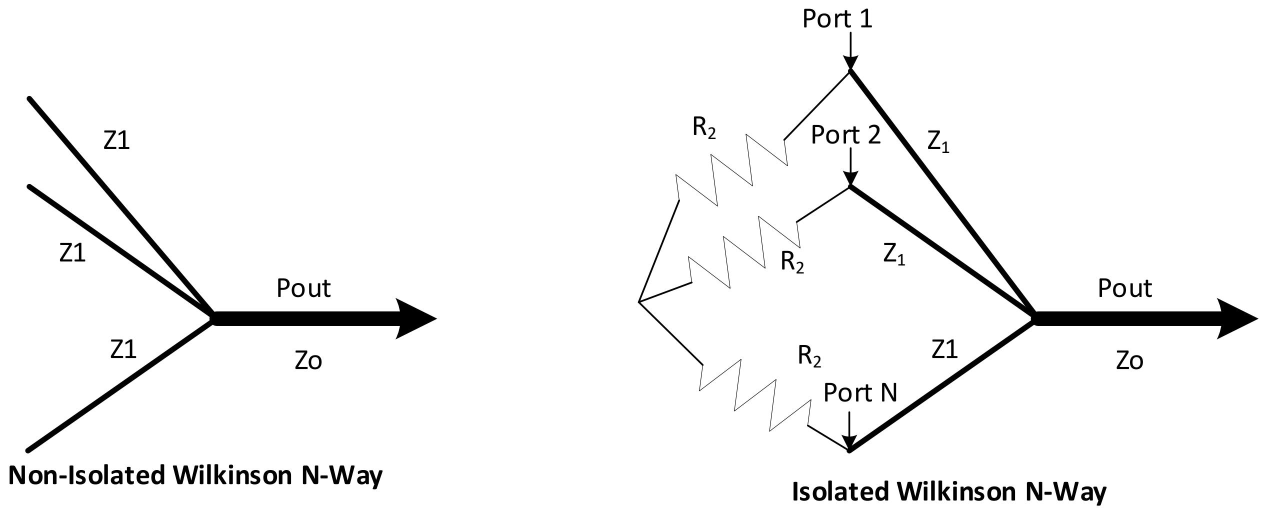

Figure 1: Wilkinson N-Way Combiners

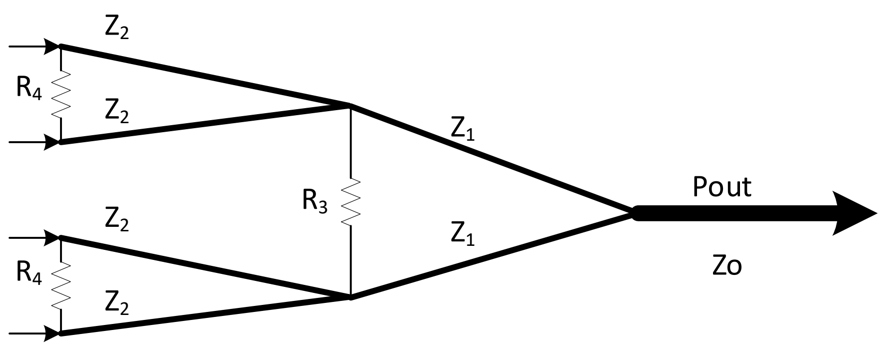

Figure 2: Multi-Section Wilkinson Binary Combiner

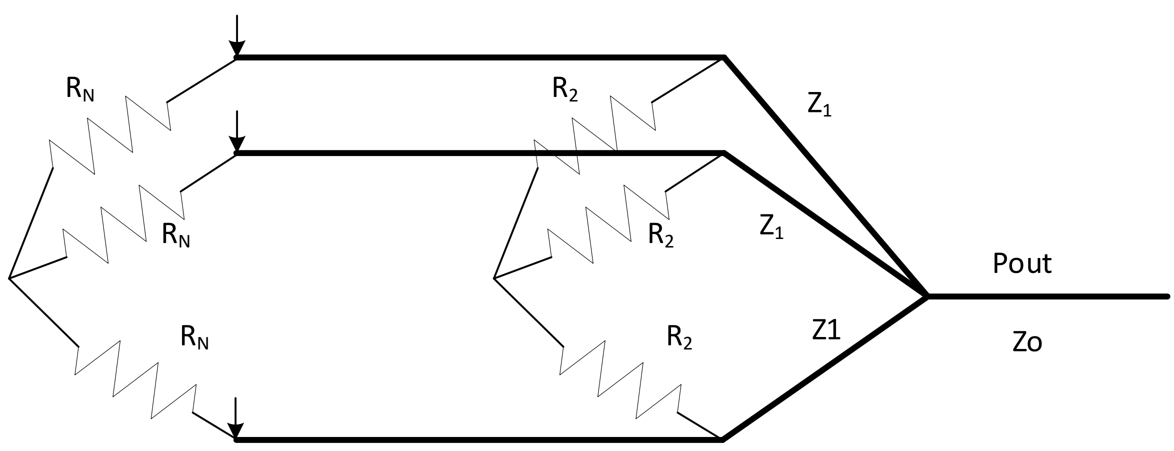

Figure 3: Multi-section N-Way Wilkinson Combiner

Explanation of Modal Analysis

Figure 4 is a mathematical representation of an N-Way circuit. There are N input ports (numbered 1 to N) and one output power numbered (N+1). At each port, j=1:(N+1), there is a forward voltage, V_{Fj}, and a reverse voltage, V_{Rj}. The inputs are V_{Fj} for j=1:N, and the output is V_{Rj} for j = N+1. If we include the load impedance at port, N+1, into a new Scattering matrix, [S'], that is N x N, then the inputs are V_{Fj}, j=1:N. We can represent the voltages at the inputs as a column vector, $V_F]$, which is simply:

$$\left. \begin{matrix} V_{F1} \\ V_{F2} \\ \bullet \\ \bullet \\ V_{FN} \end{matrix} \right]$$

Because the N-Way circuit is represented by an NxN square matrix, mathematically, the inputs can be represented by eigenvectors and there is a mathematical algorithm to derive these. However, it is much easier and more useful to determine these by understanding the geometry of the N-Way circuit. For this article, we can consider the eigenvector a mode of the circuit. For any N-Way circuit there are N modes, $Vm_j]$, j=1:N where each mode is an N value column vector. A key property of the modes in that they are orthogonal to one another:

$Vmj]^{\dagger} * Vmi] = 0$ for $j \neq i$.

Where: † is the conjugate transpose of the vector (i.e., a row vector whose values are the conjugate of the column vector values).

For all of the circuits that we discuss in this paper, we assume that they are symmetric, and that the first mode is defined as

$Vm_1 = [1, 1,..,1]^T$.

Where ^{T} is the transpose of the vector (it is easier to write a vector as a row vector and indicate it is a column vector by indicating the transpose!).

All the other modes have to be orthogonal to this mode and to each other and we can impose the condition that the 1st element of each mode has a magnitude of 1.

We can represent the input forward vector into the N-Way, $V_F]$ as the summation of these modes such that:

$V_F] = \sum a_j * Vm_j]$ j=1:N.

Where a_j is the coefficient for each forward direction of the mode.

Further, the reflection vector, V_R] can be considered as a sum of the reflected modes:

V_R] = $\sum \Gamma_j * a_j * Vm_j$] j=1:N.

Where Γ_j is the reflection coefficient for each mode.

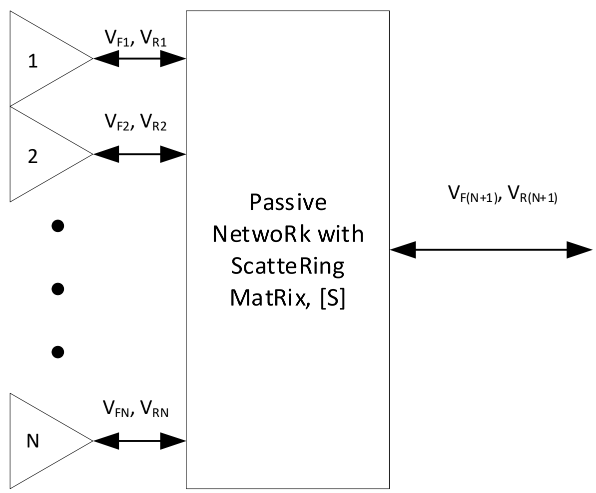

Figure 4: Mathematical Representation Of A Generalized N-Way Circuit

Simple Example of a 2-Way circuit
The simplest example is a 2-way symmetric circuit where the modes are:

Vm_1]= $[1,1]^T$ and Vm_2=$[1,-1]^T$

The common way to refer to these is an Even and Odd mode, Ve] and Vo], respectively.

Then, an input voltage to one port and not the other is given by:

$[1,0]^T$ = .5*Ve] +.5*Vo].

Analysis of a 2-Way Wilkinson with Even and Odd Modes
Figure 5 shows a two Way Wilkinson at center frequency where the 70.7 Ω transmission lines are ϴ° in length.   Figure 6 shows an equivalent circuit for operation with an even mode input. At center frequency where ϴ = 90°, the 50-ohm input impedance on each line is transformed to 100 Ω and the 50 Ω output line can be considered as two 100 Ω loads in parallel so each input is matched provided that both inputs are equal and in phase. Further, because both ports are equal voltage and phase, there is no current flowing through the resistor, so it does not affect the performance. As a result, the even mode is matched and there is no reflection.

Figure 7 shows the equivalent circuit for the odd mode where the Port 1 voltage is +ao and the Port 2 voltage is -ao, there is a virtual ground down the center of the circuit going through the junction of the two input lines with the output line. At center frequency, the lines are quarter wave. As a result, at each port, the impedance, into the transmissions, at the ports is an open. Further, because there is a virtual ground in the enter of the circuit, the 100 Ω resistor appears as two series 50-ohm resistors with a ground in between. As a result, the odd mode is matched to 50 Ω. We can write:

$V_F] = [1,0]^T = .5*Ve] +.5*Vo]$

$V_R] = [0,0]^T = 0*.5*V_e]+ 0*.5*V_o]$.

Further, the power in the even mode is (0.25/50) Watts for each input for a total of (0.5/50) Watts which all goes to the output. The power in the odd mode is also a total of (0.5/50) Watts and it is absorbed in the resistor. If there was no resistor present, then the odd mode would be totally reflected such that each port would see (0.25/50) Watts reflected.

To answer the question of how the circuit performs at other than center frequency, assume that the circuit operates at a frequency where the length of the transmission lines is 80°. Then, in the even mode, each port will operate into an impedance defined by 100 Ω transformed by the 70.7 Ω transmission line which has a reflection coefficient, Γe, calculated as:

Γe = 0.01112 – i*0.06026.

For the odd mode, each port operates into a 50 Ω resistor to ground in parallel with a ground transmission line, 80° in length. The resulting Γo, is calculated as:

Γo = -0.00387 + i*0.06211.

As a result, the reflected vector, $V_R]$, is given by:

$V_R]$= (0.01112 – i*0.06026)*.5*$[1,1]^T$ + (-0.00387 + i*0.06211)*.5*$[1,-1]^T$ = [(.00363, i*.00093), (.00750, -i*.061185]^T

Based on this analysis, the return loss at port 1 is –48.5 dB and the return at port 2 is -24.2 dB. The return at port 2 corresponds to the isolation of the combiner at the frequency where the transmission lines have a phase length of 80°.

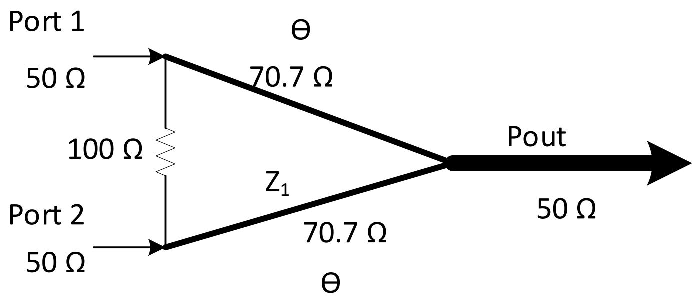

Figure 5: Circuit Representation of a 2-Way Wilkinson

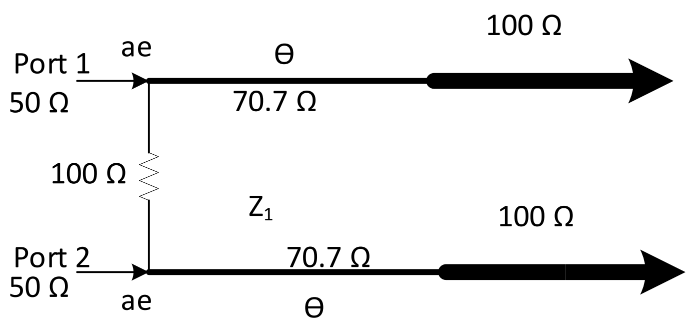

Figure 6: Even Mode Equivalent Circuit

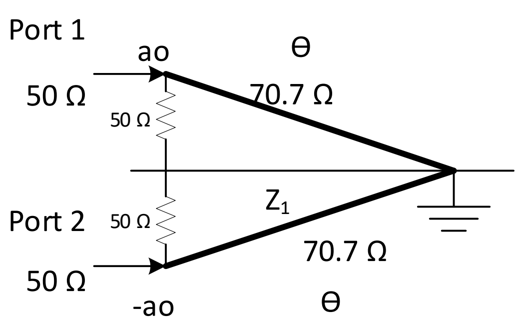

Figure 7: Odd Mode Equivalent Circuit

Modal Analysis for the Generalized Wilkinson Circular N-Way Combiner

For Wilkinson's original power combiner shown in Figure 1, The ports can be considered as arranged equally spaced around a circle and spaced (2*π/N) apart around the circle. For example, a 4-way combiner has 4 ports spaced π/2 apart around the circuit. For an N port version of this combiner, there are N modes, Vm_j, j=1:N, where each mode is given by:

$$Vm_j = [1, e^{1*i*\phi*(j-1)}, e^{2*i*\phi*(j-1)}, \bullet\bullet\bullet, e^{(N-1)*i*\phi*(j-1)}]^T \quad j=1:N$$

Where $\phi=2*\pi/N$.

For example, for the 4th mode of a 4-way circular Wilkinson combiner, the mode, Vm_4, is given by:

$Vm_4 = [1, e^{i*\pi/2*3}, e^{i*\pi/2*6}, e^{i*\pi/2*9}] = [1,-i, -1, i]^T$

In general, the modes for the 4-way are:

$Vm1 = [1,1,1,1]^T$
$Vm2 = [1, i, -1,-i]^T$
$Vm3 = [1.-1, 1, -1]^T$
$Vm4 = [1, -i, -1, i]^T$

An example of these modes being orthogonal are:

$Vm4^{\dagger} * Vm2 = [1, i, -1, -i]* [1, i, -1,-i]^T = 1-1+1-1 = 0.$

For the first mode, we can consider this the even mode, the voltages are all the same at the ports such that no current flows in the resistors. If the parallel transformed impedances at the ends of the transmission lines match the output line impedance, then the even mode is matched. An example was given for the radial combiner where the transmission lines are 50 Ω and the center output line matches to 12.5 ohms at the junction. In this case, the even mode is matched.

For the Wilkinson N-Way with the 50 Ω isolation resistors, the non-even modes are symmetric around the center of the circle such that a virtual ground exists at this point (similar to what was shown for the 2-way Wilkinson). As a result, each port operates into 50 Ω to ground in parallel with the transmission lines with each transmission line grounded at the junction. If each transmission line is a quarter-wave, then they appear as an open circuit and all the non-even modes are matched.

Consider the case of a drive to only one port. If we assume that is port 1, i.e.,

$V_F]=[1,0,0\bullet \bullet \bullet,0].$

then, it can be shown that.

$V_F] = [1,0,0\bullet \bullet \bullet,0]^T = \sum(1/N)*Vm_j]$ for j = 1:N.

We know that, at center frequency, the reflections of all the mode but the first (i.e., j = 2:N) are matched, so the only reflection is the Even mode which is matched and, therefore, there are no reflections.

For a radial combiner, where there are no resistors, the non-even modes all operate into the transmission lines terminated in a virtual ground. As a result, all operate into an input impedance jX and the reflection coefficient, Γ_j for j=2:N is given by:

$\Gamma_j = e^{-i*\sigma}$ j=2:N

Where σ = atan (2*X/Z_0).

We can then calculate the reflected power, V_R] as:

V_R] = 0*Vm1] + ∑(Γ_j /N)*Vm_j] for j=2:N

V_R] = (Γ_j *∑(1/N) Vm_j] for j=2:N.

From the expression for V_F] we can obtain:

(Γ_j *∑(1/N) Vm_j] for j=2:N = V_F]-Vm1] =$[1,0,0\bullet\bullet\bullet,0]^T$ - 1/N*$[1,1, ,\bullet\bullet\bullet,1]^T$ = $[(N-1)N, -1/N, -1/N,\bullet\bullet\bullet,-1/N]^T$

We get that:

(Γ_j *∑(1/N) Vm_j] for j=2:N = Γ_j* $[(N-1)N, -1/N, -1/N,\bullet\bullet\bullet,-1/N]^T$

Since,

$\Gamma_j = e^{-i*\sigma}$ is a constant.

Finally, for the radial combiner or the more general non-isolated Wilkinson N-Way, for a single input port, the magnitude of the reflection coefficient at port 1 is (N-1)/N and the coupling to the output ports has a magnitude of (1/N). More generally, any combination of inputs can be analyzed by determining the coefficients of the modes and multiplying the coefficients of the modes by the reflection coefficient for each mode which is straightforward since the reflection for all modes, but the even mode, are the same. As an example, a 4-way Wilkinson excited on the first 2 of the 4 ports can be modelled as:

V_F] = $[1,1,0,0]^T$ = ∑a_j*Vm_j] for j=1:N.

Assuming the 4 modes for the 4-Way combiner,

Vm1 = $[1,1,1,1]^T$
Vm2 = $[1, i, -1,-i]^T$
Vm3 = $[1.-1, 1, -1]^T$
Vm4 = $[1, -i, -1, i]^T$

We can form the matrix, [M], consisting of the columns of Vm
[M]=

```
   1.0000 + 0.0000i   1.0000 + 0.0000i   1.0000 + 0.0000i   1.0000 + 0.0000i
   1.0000 + 0.0000i   0.0000 + 1.0000i  -1.0000 + 0.0000i   0.0000 - 1.0000i
   1.0000 + 0.0000i  -1.0000 + 0.0000i   1.0000 + 0.0000i  -1.0000 + 0.0000i
   1.0000 + 0.0000i   0.0000 - 1.0000i  -1.0000 + 0.0000i   0.0000 + 1.0000i
```

Then, the column vector a] =[a_1, a_2, a_3, a_4]^T is defined by:

M]*a] = [c].

Where [c] is a single or multiple column vectors with the desired inputs for each port.

Then we can solve for a\ by multiplying by the inverse of [M]

$[M]^{-1}*[M]*a]^T = a]^T = [M]^{-1}*[c]$.

Generally, we can find the answer to all combinations by setting [c] equal to all those possibilities and multiplying by $[M\}^{-1}$.

We can then obtain an answer matrix [a] where the columns are the coefficients for the different combinations. As an example, we can set up a [c] matrix:
[c] =

1	1	1	1	2
0	1	1	1	0
0	0	1	1	1
0	0	0	1	0

Where each column of c is a distribution of power. For instance, the first column is the case of a single input into Port 1, and the following 3 columns are inputs into an increasing number of ports. Finally, the last column is an arbitrary distribution of 2 volts into Port 1 and 1 Volt into port 3. The corresponding coefficients to the modes are:

[a] =

0.2500 + 0.0000i	0.5000 + 0.0000i	0.7500 + 0.0000i	1.0000 + 0.0000i	0.7500 + 0.0000i
0.2500 + 0.0000i	0.2500 - 0.2500i	0.0000 - 0.2500i	0.0000 + 0.0000i	0.2500 + 0.0000i
0.2500 + 0.0000i	0.0000 + 0.0000i	0.2500 + 0.0000i	0.0000 + 0.0000i	0.7500 + 0.0000i
0.2500 + 0.0000i	0.2500 + 0.2500i	0.0000 + 0.2500i	0.0000 + 0.0000i	0.2500 + 0.0000i

Where the 1st column of [a] is the coefficients for the case of just the input into Port 1 and are equal to the previously derived value of 1/N which, in this case, is 0.25. The last column:

a] = [.75,.25, .75, .25]^T

corresponds to the coefficients for the arbitrary input [2,0,1,0]^T

Of course, [c] can be any combination and the matrix manipulation will result in an answer.

Analysis of a Multi-Section Binary Wilkinson

With reference to Figure 2, there are N ports where N is a multiple of 2. Once again, there are N modes. However, the ports are in line, i.e., not circular. So, for a 4-Way version, as shown in Figure 2, the 4 modes would be:

$VM_1] = [1,1,1,1]^T$
$VM_2] =$ **$[1,-1,1,-1]^T$**
$VM_3] = [1,1,-1,-1]^T$
$VM_4] = [1,-1,-1,1]^T$

For an 8 way, a similar version can be developed that accommodates the geometry of the combiner.

As was the case in the prior examples, the single input is given by:

$[1,0,0,0]^T = 1/4*\{ Vm_1] + Vm_2] + Vm_3] + Vm_4]\}$.

In order to match the modes, other than the even mode, it can be seen that following must be true:

- For modes, $Vm_2]$ and $Vm_4]$, it can be seen that resistor, R4, must be equal to twice the port impedance. For an input impedance of 50 Ω, R4 must be 100 Ω.
- To match $Vm_3]$, resistor R3 must be twice the impedance after transmission line Z1.

Once the other modes are matched, the mode, Vm1] is matched by impedance matching the equivalent outputs to the input impedance to the common output line.

The coefficients for the modes can be derived via the procedure shown in the previous section.

Modal Analysis of Multi-Section N-Way Combiner

With reference to Figure 3, the analysis of the N-Way Wilkinson remains but the modes, $Vm_j]$ for j = 2:N, operate into a match that consists of multiple resistors. Figure 8 shows the equivalent circuit for a single port in the even mode. Because the even mode voltages are all the same around the structure, no current flows through the resistors so they are not part of the circuit. The circuit is then a multi-section impedance match to the load which has a virtual impedance of N*Ro where Ro is the output impedance at the junction to the transmission lines. By using a multi-section match, the even mode impedance match can be extended over a broad bandwidth.

Figure 9 shows the equivalent circuit for the non-even modes. Each of them operates with a virtual ground down the center of the structure such that each of these modes operates with the resistors to ground. Since the characteristic impedance of the matching line is selected to optimize the even mode match, the resistors are then chosen to optimize the match to the non-even modes. While there is a theoretical method to do so, it is easier and quicker to use a CAE program such as MWO or ADS to quickly optimize the resistor values for best match over a given bandwidth.

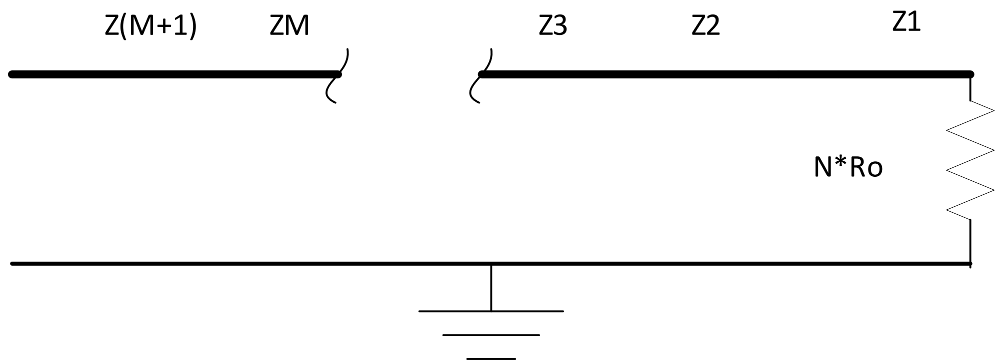

Figure 8: Equivalent Circuit of an Even Mode Single Port of an N-Way Multi-Section Combiner

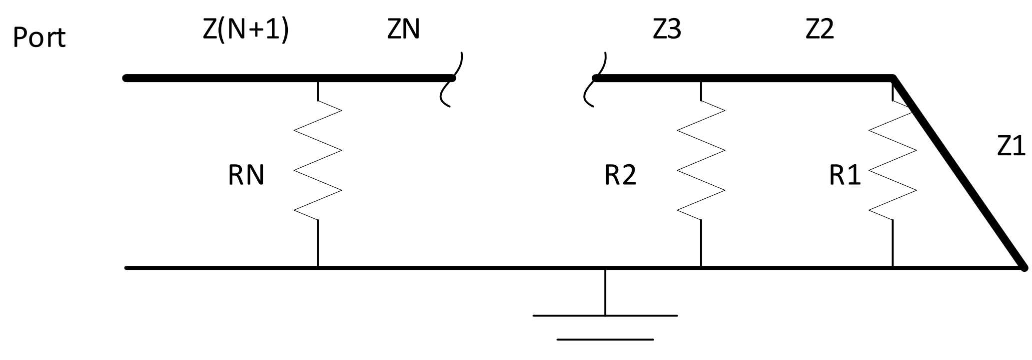

Figure 9: Equivalent Circuit of a Non-Even Mode Single Port of an N-Way Multi-Section Combiner

Figure 10 shows the equivalent Even Mode circuit for a 3-Way combiner operating 200-900 MHz. In the even mode, the resistors are not part of the circuit so do not have to be included in the circuit design. Figure 11 shows an equivalent circuit for the non-even modes where there is a virtual ground at the center of the structure such that only circuitry to that virtual ground need be considered. Shown is a 2-stage resistive matching. Figure 12 shows the insertion loss of this circuit which is, ideally -4.77 dB. Finally, Figure 13 shows the match for both the even mode and the non-even modes. The nominal return loss, in the non-even modes, indicates that the isolation will be about 15 dB.

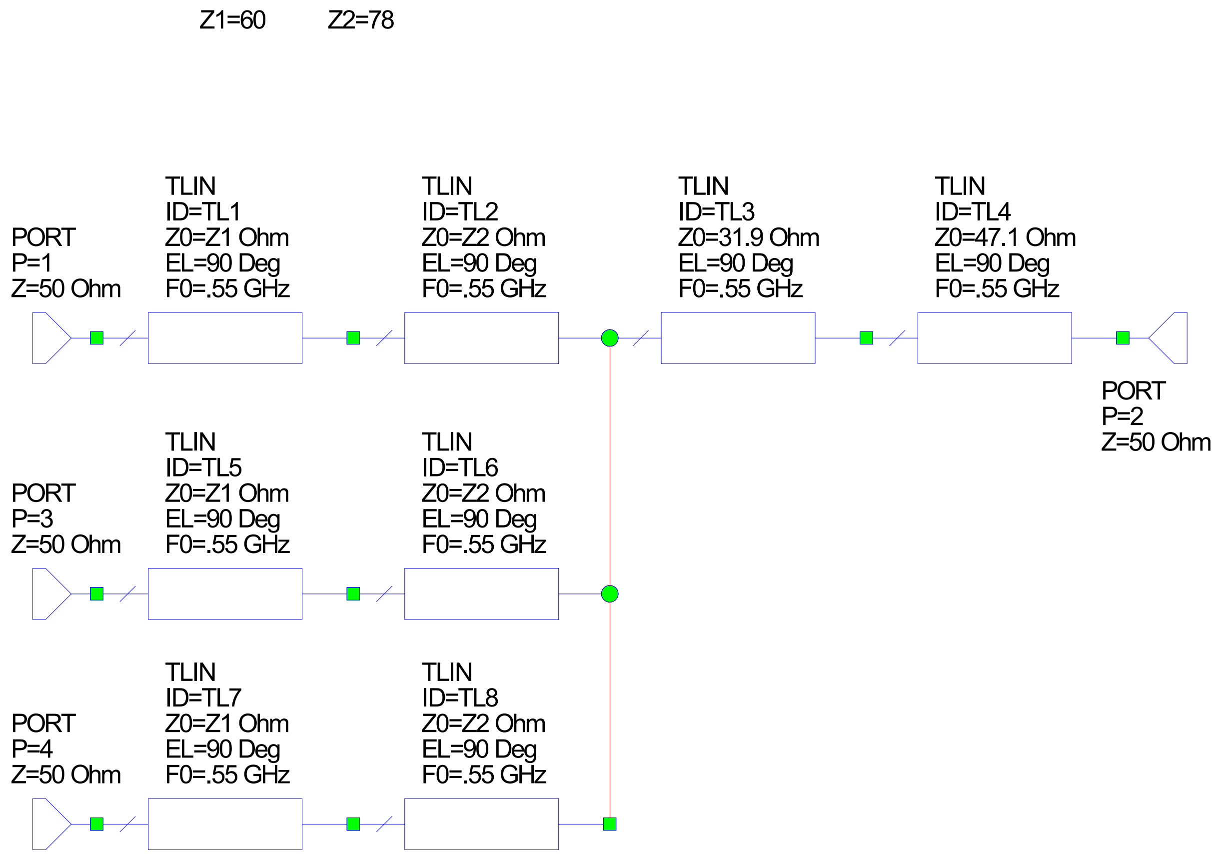

Figure 10: Part of circuit for 3-Way combiner 200-900 MHz

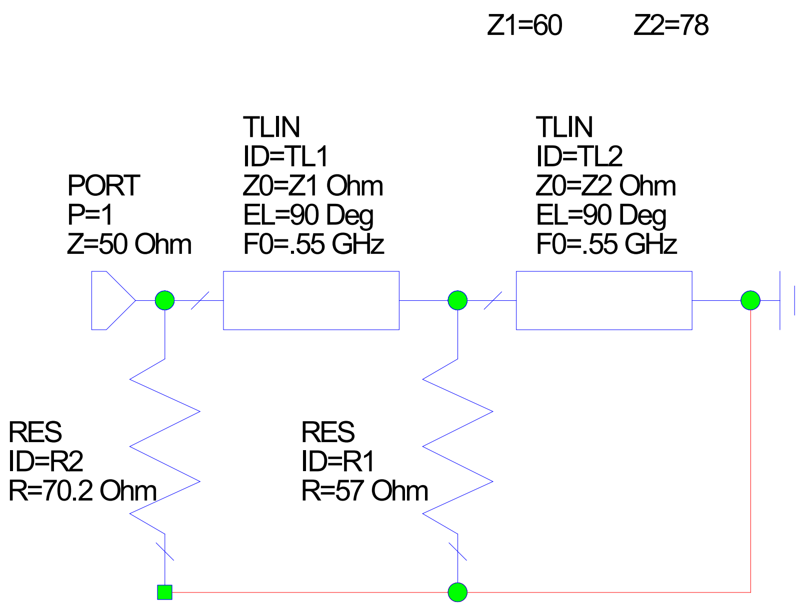

Figure 11: Schematic of 3-Way for Non-Even Modes

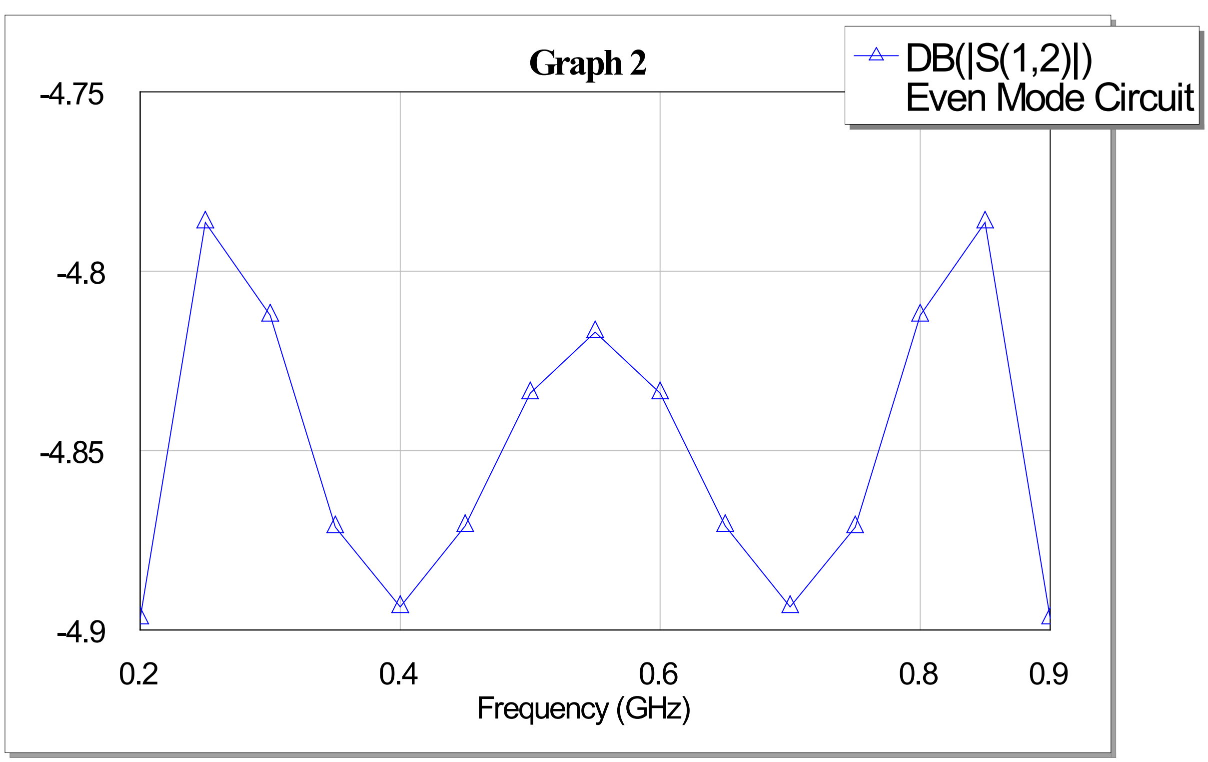

Figure 12: Insertion Loss of 3-Way for Even Mode

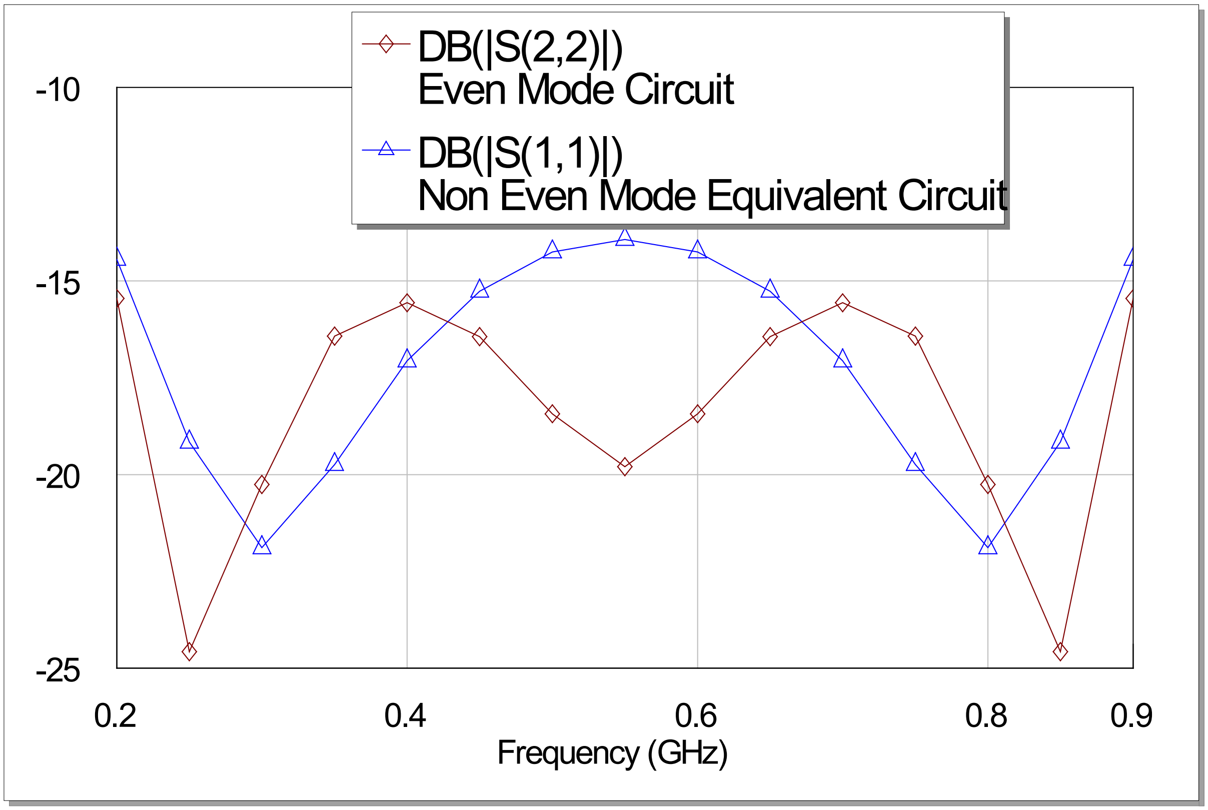

Figure 13: Return Loss for Both Even and Non-Even Modes

Summary

This article has shown that determining the modes for an N-Way combiner allows an easy and intuitive way to understand , analyze, and design these circuits. A mathematical algorithm for determining the coefficients has also been presented. In future articles , it will be shown how use of modal analysis will help improve performance, especially insertion loss of isolated combiners.